An engineering firm wants to study the effect of tire brand (A, B, C, D) on tread wear. They measure tread wear (in thousandths of an inch) after 1,000 miles.

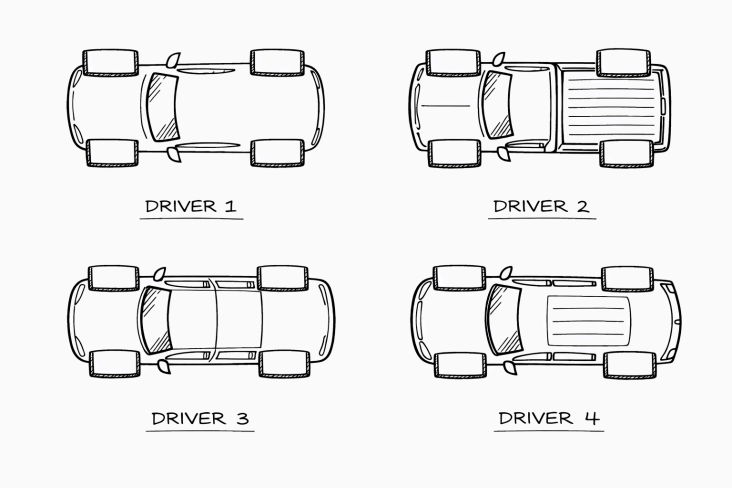
1. What is the response variable in this study?
2. What is/are the experimental factor(s)? What are the treatment levels?

Design I

They first propose a study where they recruit four drivers with different cars to each drive a randomly assigned brand of tire for 1,000 miles.

3. What are the experimental and measurement units in this study? How many of each are there?

4. Label each of the tire boxes below with one possible random assignment of the four brands.



5. In the space below, sketch a data frame that could represent the observed data from this study.

6. What test statistic would you use to measure the effect of tire brand on tread wear in this design?

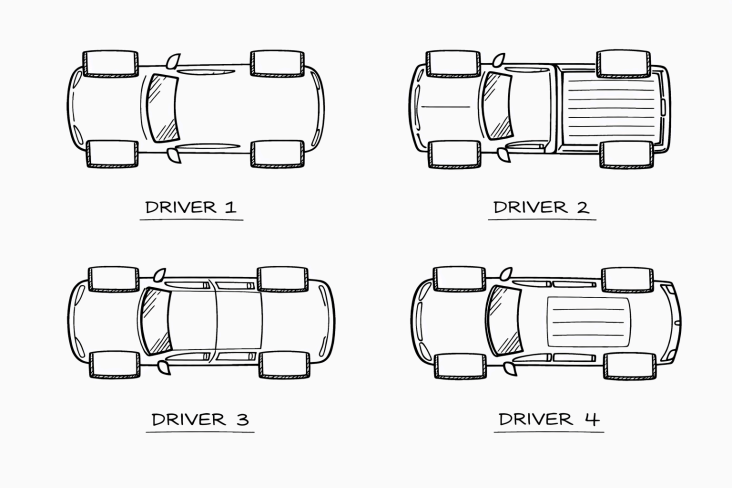
7. Beyond the effect of brand, what other factors could contribute to variation in tread wear in this design? List at least three.

Design II

To control for some of the nuisance variation, they instead propose a second design where each driver uses all four brands of tires, but the brand assigned to each tire position (front-left, front-right, rear-left, rear-right) is randomized within each driver.

8. What are the experimental and measurement units in this study? How many of each are there?

9. Label each of the tire boxes below with one possible random assignment of the four brands.



10. In the space below, sketch a data frame that could represent the observed data from this study.

11. Write out a linear model that could describe the effect of tire brand on tread wear in this design. Define every term clearly, including what the indices $j(i)$ and $k(i)$ represent in this context and what the values of J, K, and n are.

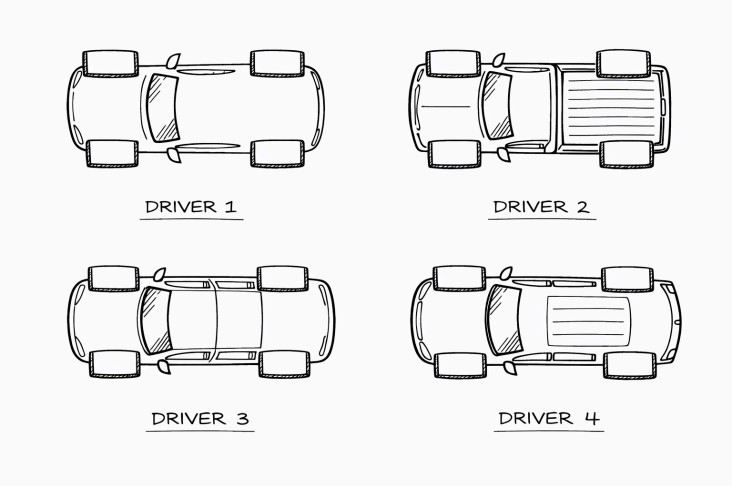
12. What is the difference between the manner in which you would calculate your test statistic for the effect of brand in this design compared to the first design?

Design III

The engineers note the tire wear on front tires is often different than rear tires, and that the wear on the left side of the car is often different than the right side. To control for this variation, they propose a third design where each driver uses all four brands of tires, but the brand assigned to each tire position (front-left, front-right, rear-left, rear-right) is randomized within each driver with care taken that each position is represented equally across brands.

13. What are the experimental and measurement units in this study? How many of each are there?

14. Label each of the tire boxes below with one possible random assignment of the four brands.



15. In the space below, sketch a data frame that could represent the observed data from this study.

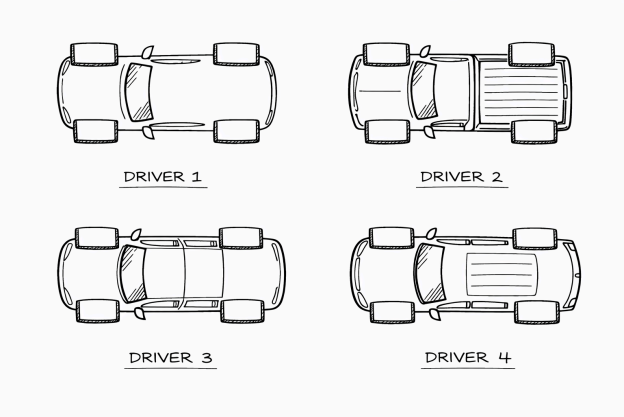
16. Write out a linear model that could describe the effect of tire brand on tread wear in this design. Define every term clearly, including what the indices $j(i)$, $k(i)$, and $l(i)$ represent in this context and what the values of $J, K, L,$ and n are.

17. Say that after calculating the appropriate statistic for the effect of brand in this design (and estimate of the parameter above), you find it's non-zero. A skeptical engineer on your team suggests that effect might have just been due to random chance. Explain precisely where the randomness comes from in this design.

Design III, continued

18. Using the data frame you sketched for Design III, sketch out below left the Latin Square that random assignment corresponds to. Be sure to label the rows and columns appropriately. Then, on the right, sketch out a second Latin square that is one that you *could* have obtained from the randomization procedure, but is different than the first one.

19. Fill in the tire boxes below with the labels that correspond to the second latin square you sketched above.



20. On the back of this sheet of paper, sketch a diagram that illustrates the effect that these different designs will have on the standard errors of the test statistics for the effect of brand.

    It should contain all of the following elements:

    - The x-axis should be labelled T for generic test statistic.
    - The y-axis should be labelled "density".
    - Sketch six total density curves: the null distribution and the alternative distribution for each of the three designs. Label which curve corresponds to which design and which distribution.
    - Add annotations for the rejection regions.
    - Shade the appropriate areas to illustrate the power of each design.